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An Energy Systems View of Karl Marx's Concepts of Production and Labor Value

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ABSTRACT

This paper explores models of production by Karl Marx and other authors using microcomputer simulation to relate resources and dynamic process. Energy systems diagrams are adopted both to translate many concepts without semantic misunderstanding and to facilitate their pertinent computer simulations. Several models of production and labor value are discussed, including donor-receiver concepts of value, the labor donor concept of value by Marx, Marx's model of surplus value distribution, capital and price. A dynamic simulation model of Marx's concepts is developed. Finally, Marx's views are explored using other authors' interpretations of his concepts.

INTRODUCTION, ENERGY SYSTEMS AND THE MODELS OF MARX

Now in the new millennium changes in the economy and its relationship to environmental resources, growth, and social welfare raise questions about which measures of value are appropriate for public policy. As part of this reexamination, there have been studies of the economic concepts of the 18th and 19th Century, especially those of Karl Marx. For expressing discussions concisely and making models concrete, translation with energy systems diagrams clarifies semantic confusion, facilitates mathematical expression, emergy evaluation, and computer simulation.

Some basic concepts of modern economics have been expressed with energy systems methods in previous publications (Odum, 1983, Chapter 23; Odum, 1987, 1996; Scienceman, 1984, 1995). Scienceman (1989) uses a summary of the history of economic values by R.L. Heilbroner to discuss concepts of value and capital in the history of economics, comparing it with emergy concepts (emergy is spelled with an "m"). In this paper, with energy systems methods, we examine models of production by Marx and others using microcomputer simulation to relate resources and dynamic process. When the Marx model is expressed in energy systems language, related to resources and degrees of feedback, simulated, and compared with other models, additional clarity may result on its applicability to the 19th, 20th, and 21st Century.

Basic System of Production and Consumption

Figure 1 illustrates the basic system of production and consumption of an economy as expressed in energy systems language symbols. The circulation of money from consumers to producers and back begins most elementary economics textbooks. The energy diagram also shows explicitly the flow of produce to consumers and services to producers that are bought with the money payments and are a counter current to the flows of money. Materials are incorporated in production and released as waste and by-products and dispersed for recycling to production again.

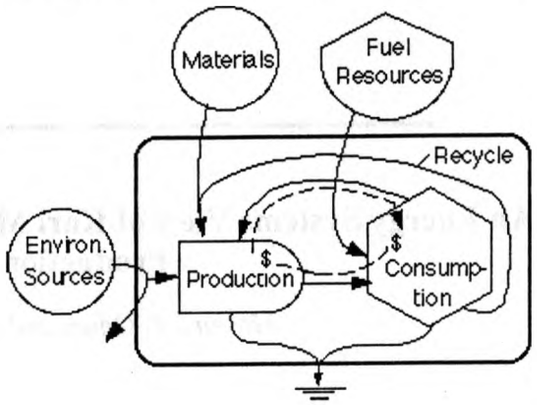


Figure 1. *Energy systems diagram of the basic plan of the economy with production, consumption, and its dependency on environmental resources and fuel resources including the circulation of money as a counter current.*

The realities of resource limitations are also shown. The potential energy inflows making the economy possible are shown entering from the sources (circles), while the degraded used energy (that cannot be reused) is shown leaving the system through the pathways to the "heat sink" at the bottom of the diagram. The materials are recirculated within the system, and those lost outside the system have to be replaced from the sources along with the fresh inflows of potential energy.

Growth Limits by External Availability of Energy Sources

There is not much controversy about Figure 1 until questions are raised about whether the external inputs can be limiting. Economic views often assert that there is always another resource to be substituted in the long run for the ones that technology develops for their use. Ecoenergetics views resources as substitutable with the aid of technology, but asserts that substitutions and technologies themselves require use of rich resources so that growth is ultimately resource limited.

Figure 1 can represent Ricardo's early view of natural value as limiting with his example of a corn economy (Wolff, 1984). As shown, environmental inputs are used to produce products for consumers such as corn. The consumers are people that provide the services to corn production. When natural inputs change, so does the buying power and hence prices due to inflation or deflation. An increase in environmental input causes an increase in value--thus profits.

The diagrammatic language distinguishes between the two resource hypotheses as shown in Figure 2. Here Figure 2a has availability constant, regardless of use, because of substitutability (usual economic viewpoint). Mathematically this is expressed as energy availability constant (E = constant, Figure 2b). The opposite point of view is expressed with Figure 2c. Here available resources, even when freely substituted, are ultimately dependent on externally determined inflows--some like sunlight are abundant and rapid, whereas others like fuels are generated slowly by the biogeochemical cycles of the earth.

The ways of connecting pathways force the diagrammer and the readers to consider the energy and material constraints. The items on the left are those that are abundant but spatially dispersed, whereas the items on the right are more concentrated, requiring the convergence and resource use during transformations for their support. Thus, the diagram shows energy hierarchy found in all systems.

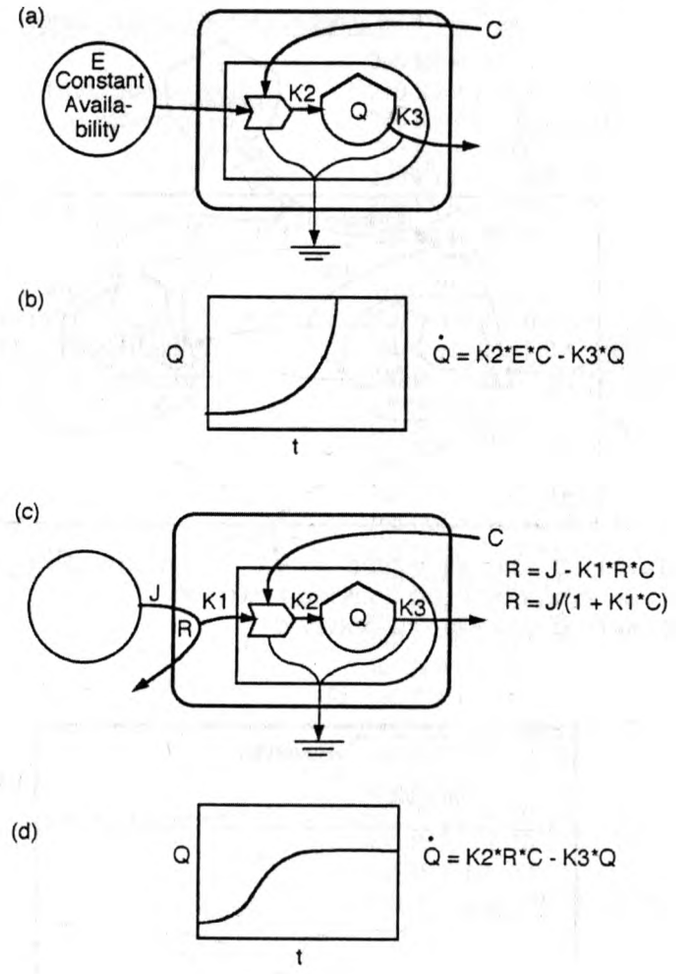


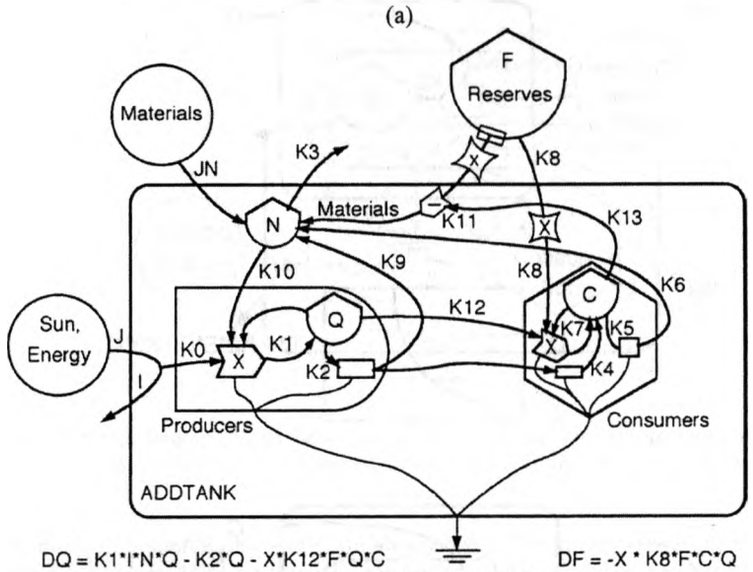
Figure 2. *Energy diagrams and microcomputer simulations for representing two competing concepts regarding sources and economic growth. (a) Constant and therefore unlimited availability of resources supporting the economy; (b) unlimited, accelerating growth based on constant availability of resource basis; (c) availability of resources dependent on an externally controlled inflow; (d) growth eventually limited by external rate of replenishment of available reserves.*

The diagrammatic language has its mathematical equivalence that may be expressed in equation forms, which are simulated producing graphs of variables with time. Equations in Figure 2b and 2d are for the systems in Figure 2a and 2c. Simulation of these equations generated the graphs in Figure 2. By means of the diagrams, equations, and simulation graphs, concepts about resources and growth are given precision.

Simulation of an Aggregated Model of Production and Consumption

Given in Figure 3a is a model, ADDTANK (Table 1), with production, consumption, material recycle and both kinds of energy sources, the steady renewable input on the left and the storage tank

(a)



DQ = K1*I*N*Q - K2*Q - X*K12*F*Q*C DF = -X * K8*F*C*Q

DN = JN - K3*N + K6*C + K9*Q - K10*I*N*Q + X*K11*F*C

DC = K4*Q - K5*C + X*K7*F*C*Q - X*K13*F*C

(b)

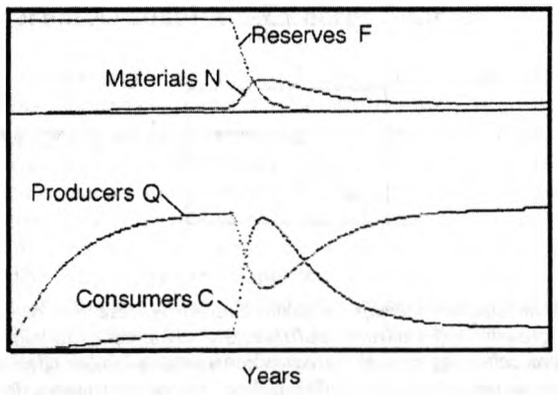


Figure 3. *Model of the dynamic relationships between producers, consumers, and the cycle of materials. (a) Energy systems diagram and equations; (b) simulation of growth of the systems in model in (b) using the BASIC program ADDTANK in Table l. Growth is first based only on environmental sources. Use of non-renewable organic reserves is started halfway across the screen.*

of organic reserves on the right. The model applies similarly to ecological systems on a small scale and to the economy in Europe during the 18th and 19th Centuries.

As the simulation is run in Figure 3b, growth first occurs without use of the organic reserves. In a lake, plant producers develop followed by growth of the dependent consumers. Then halfway across the screen consumers start to use the reserve storage with a surge of growth that also pulls down the producer assets and generates a surge of waste product materials to be assimilated.

Table 1. BASIC Program ADDTANK for simulating model in Figure 3.

```
10 REM IBM
20 REM ADDTANK (Autocatalytic prod.
on renewable source; later consumers on
reserves)
30 CLS
40 SCREEN 1,0:COLOR 0,0
45 KEY OFF
50 LINE (0,0)-(319,180),3,B
70 LINE (0,55)-(320,55),3
72 LOCATE 2,1:PRINT "Reserves"
74 LOCATE 6,1:PRINT "Materials"
76 LOCATE 14,1:PRINT "Producers"
78 LOCATE 20,8: PRINT "Consumers"
80 X = 0
90 C = 10
92 N = 100
95 DT = 20
100 Q = 10
110 F = 10000
130 F0 = 200
140 Q0 = 15
150 N0 = 100
160 C0 = 10
170 T0 = 30
190 JN = .005
200 K0 = 8.999999E-03
210 K1 = .0001
220 K2 = .001
230 K3 = .02
240 K4 = .0001
250 K5 = .001
260 K6 = .05
270 K7 = 1E-09
280 K8 = 1E-08
285 K9 = .00001
290 K10 = .000001
300 K11 = 5.2E-8
305 K12 = 1E-09
307 K13 = 1E-7
310 J = 100
320 IF J < 0 THEN J = 0
330 I = J / (1 + K0 * N*Q)
340 DC = K4*Q - K5*C +X*K7*F*C*Q -
X*K13*F*C
350 DQ = K1 * I*N*Q - K2 * Q -
X*K12*F*Q*C
360 DF = -X* K8*F*C*Q
365 DN = JN - K3*N + K6*C +K9*Q -
K10*I*N*Q +X* K11 *F * C
370 Q = Q +DQ*DT
380 C = C + DC*DT
390 F = F + DF*DT
400 N = N + DN*DT
405 T = T + DT
410 IF N <.00001 THEN N = .00001
420 PSET (T/T0, 180 - Q/Q0) ,1
440 PSET (T/T0, 55-N/N0),2
450 PSET (T/T0, 180-C/C0)
460 PSET (T/T0, 55 - F/F0),2
465 IF T/T0>130 THEN X = 1
470  IF T/T0 <320 GOTO 300
```

The model provides perspective on economic conditions in Europe with mostly an agrarian society operating on steady renewable resources in the 19th Century followed by explosive growth of an economy increasingly based on fuel resources accelerating in the 19th Century.

In summary, Figures 1-3 illustrate the way ideas of economics can be expressed in several objective, alternative ways including models diagrams showing pathway relationships and storages, energy laws and energy hierarchy, mathematical equations, and simulation graphs.

The diagrams are readily understood by most people, allowing those who use mathematical concepts to discuss concepts with those who think verbally, without semantic misunderstanding.

Donor-Receiver Concepts of Value

Martinez-Alier (1987), with a book entitled Ecological Economics, reviewed the work of a dozen authors whom he classified as pioneer contributors to ecological energetics in which value is derived from the input of resources (climate and land), measured with physical measures of these basic inputs.

The ecoenergetics roots attempt to derive and measure value in units of inputs of natural resources from outside the economy. Symbolized by the face on the left in Figure 4c, value is "donor"

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defined from the biosphere, in particular from the agricultural sector. Those with this resource concept of value are sometimes called Physiocrats. The evaluation of resource contributions with the new measure, emergy, spelled with an "M," is an attempt to put these evaluations on a scientific basis.

Shown in Figure 4a is another donor-type value concept. With Marx's Labor value concept, the contribution of everything downstream is determined by the hours of labor embodied in the pathway.

As viewed by economics of the 20th Century, the most common definition of economic value is "what humans are willing to pay as in markets." It is the market value to an individual person or firm. It is determined by the consumer in determining price.

The human perception value concept is represented in Figure 4b by the human figure in the box marked "consumers." Value is defined by the selfish, local, individual, momentary perception as to what is good for the individual collectively generating demand. This is a "receiver" defined value.

Thus, for over a century there has been a deep division of thought on value that is expressed in Figure 4. Figure 4a shows a donor value starting with labor passing along pathways of work. Figure 4b shows consumers controlling market price, thus expressing their receiver-based value. Figure 4c shows a more complete picture. Donor value of environmental contributions, measured in emergy units, flows from the left. Prices result from the amount of money circulating relative to the product use.

Economic Designs for Maximum Empower

The external donor resource concept of value that goes back to Ricardo and before (Martinez-Alier, 1987) has been given rigorous definition by defining emergy as the available energy of one form necessary to develop that of another form (when *accounted for by means of the rules of Emergy Algebra*). By developing tables of the solar emergy per unit energy (solar transformity), a practical means has been developed for calculating donor value. The maximum power principle of Boltzmann and Lotka and others, is more rigorously stated as the principle of maximum rate of useful emergy use (empower) as follows:

> The economy that emerges and prevails during self organization is the one that maximizes its rate of useful emergy use.

In terms of the system shown in Figures 1, 2, and 4c the solar emergy use is determined from the sources from outside plus the use of nonrenewable storages inside. The systems designs that emerge are those where production is fed back to reinforce the gaining of new sources and the use of these with optimum efficiency consistent with maximum performance. Whereas this paper does not concern emergy evaluation, we do include this paragraph as the justification for the characteristic designs used to represent economies of the past and present. (For more on emergy, see Odum, 1986a, 1986b; Scienceman, 1986; Odum et al., 1987, 1996).

Labor Donor Concept of Value of Marx

The value concept of Marx fits neither the external resource donor (emergy) nor internal receiver concepts in Figures 4b and 4c. As sketched in Figure 4a, value of exchange is derived from the human service contributed to material in commodity exchanges. It is value determined by a donor process, but the donors are the people within the system. It's as though value is a fountain that emerges within workers. Marx acknowledges the necessary incorporation of materials, but these do not accrue labor value until incorporated through a labor-facilitated production process. When diagrammed as in Figure 4a, a network of labor value flow results which does not deal with inputs of environmental resources or information.

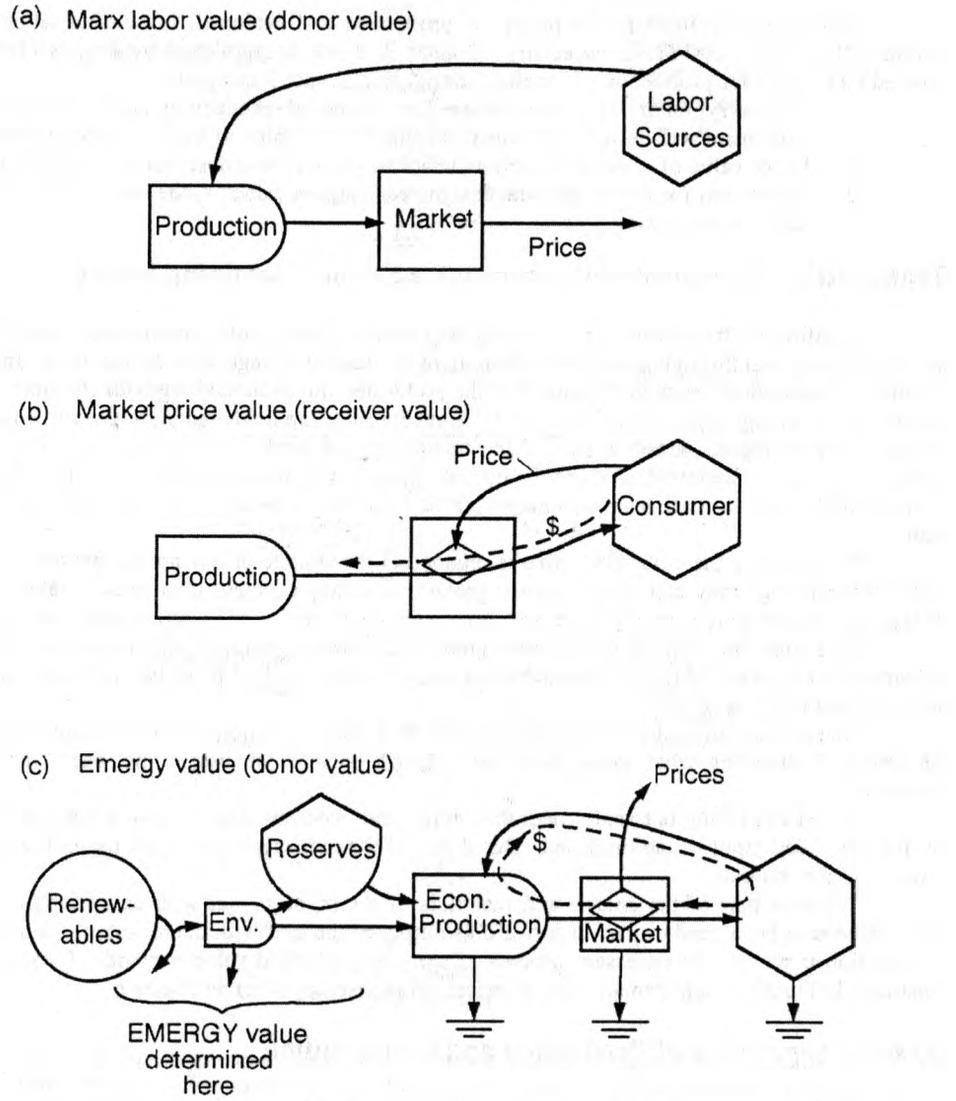


Figure 4. *Illustration of donor and receiver definitions of value. (a) Labor value concept of Marx with value coming from human hours contributed; (b) market price value determined by receiving consumer's willingness to pay money; (c) emergy value derived from the resource contributions.*

Capital

The flows of value were called flows of capital by Marx, with meaning quite different to modern uses of the word. See Table 2 from Eagly (1973, p. 131). As pointed out by Eagly, the word capital, as used by Marx, was a term for all kinds of flow of economic value, a more general use of the word than its modern use. In modern use, capital often refers to the values in large units such as buildings, equipment, infrastructure, etc.

Capital was defined as the means of production and evaluated in units of labor time. In volume III of DAS CAPITAL especially, Chapter 3, Marx as published by Engels (1894; 1962) divided labor basis for production of "capital," designated C, into 3 categories:

1. Necessary labor time to produce the means of production such as structures and equipment was designated "constant capital," c. Capital to Marx is a social relation.
2. Labor value of products distributed back to laborers was designated "variable capital," v.
3. Production for capitalists was designated "surplus value" s, the excess derived from the variable production process.

Transmitting Embodied Power through a Sector Containing Storage

Confusion often arises in considering the numerical output of a donor based, embodied value in being transmitted through a sector in which there is a capital storage with depreciation. In Figure 5, 100 units of embodied input are flowing into the sector per unit of time along with 100 units of energy and materials being used by that sector. In Figure 5a the input of materials goes to maintain the storage, some being dispersed as part of the transformation work and depreciation. The embodied value, however, is transferred 100% to the output. In such a transmission the output has a higher ratio of embodied value to materials and energy. This principle is clearly given by Marx for his labor value.

In Figure 5a the embodied value is retained in the storage or transmitted beyond. Embodied value in the storage may increase if there is growth, or it may decrease if there is storage decline. If storage is constant there is steady state, and input embodied value equals output embodied value.

In Figure 5b, half of the product goes to a growing storage (+25/unit time), and half is transmitted beyond to the right. The embodied value is split. Each of these two pathways carries half of the embodied input of 100.

Marx's examples allowed for growth, but he adjusted his numbers to a steady state through the device of diverting what would have been growth outside, labeling it "not used," as shown in Figure 5c.

When something is transformed, the energy and materials may be less in the output product, but the embodied input is undiminished. But if part of the output product is not used, its embodiment is not passed to outputs.

Whereas part of the energy and materials of the input go out with depreciation, embodied labor value is either stored or passed forward as output of the unit with depreciation. In other words, depreciation is part of the necessary process of carrying embodied value forward. These principles, illustrated in Figure 5, help explain the numerical values used by Marx in Figure 6.

Seasonal Separation of Production and Consumption

With the economy of Europe still highly agricultural in the last century, it was customary then to think of a sharp seasonal pulse of production which went to markets. Then, as a separate stage, the market products were distributed to workers and capitalists. Marx deals with the first with production models numerically evaluated in units of labor time that include the direct paid and indirect unpaid work necessary to the production process. Then the distribution of the products is traced with numerical values of flows with monetary evaluations of "prices" for exchange.

Marx's Model of Surplus Value Distribution

The model of production in terms of labor values discussed by Marx in his volume III, Chapter 3, is diagrammed in energy language notation in Figure 6. Notice two main production processes (within producer symbols). The one above has a storage (tank symbol) representing long lasting structures that have depreciation.

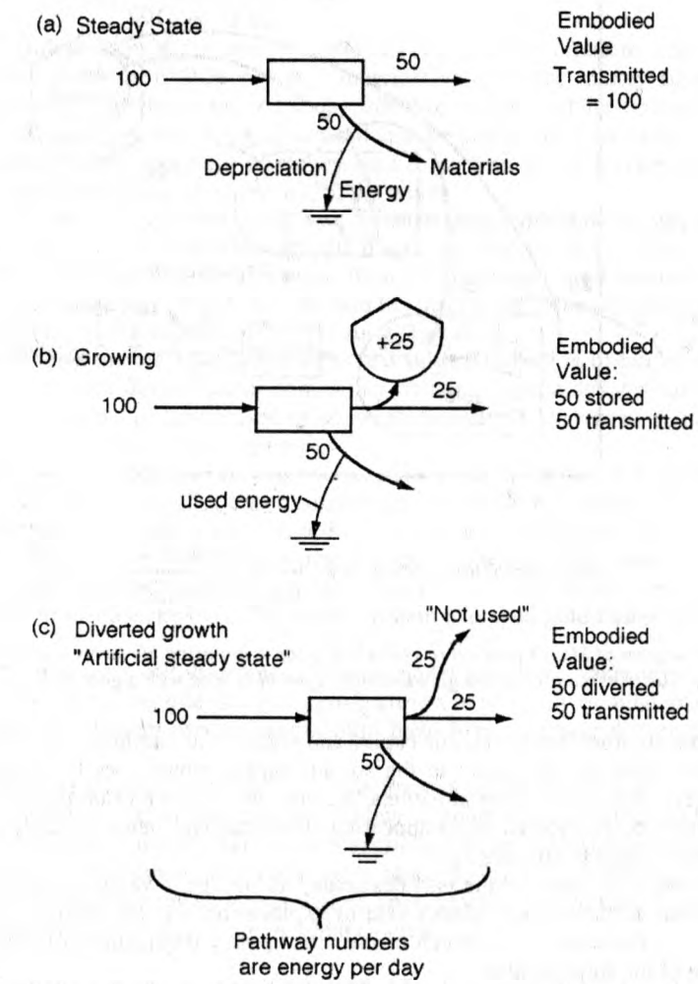
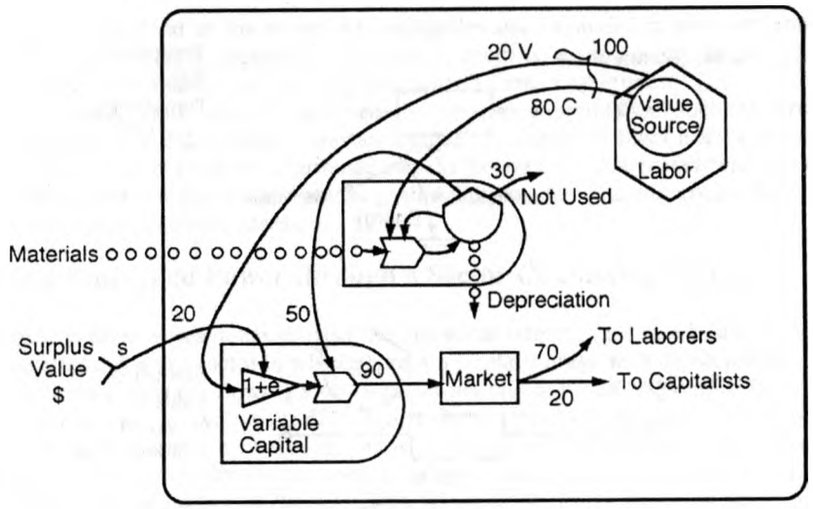


Figure 5. *Concept of donor-type value transmitting the embodiment of an input through a transformation to an output (applicable to both Labor value and EMERGY Value). Numbers on the pathways are energy flows in steady state. (a) Steady state; (b) growing situation with net increase within the storage tank; (c) a growing state represented as a steady state by diverting as "not used" what would have been growth increase.*

To explain concepts, Marx supplied some numbers normalized as 100% of the work contribution of the laborers. In Figure 6, the Marx concepts of production, its source of value from labor, and the destination of the value in the products of labor and their fate are diagrammed.

In this example (from Marx, p.153) the total labor-determined value of the inputs to two production processes was given as 100 per year of which 80 went by the lower pathway into the constant capital means of production. This unit was not in steady state, since 30 is shown not used and 50 is the output labor value going to the lower production process.

The two inputs to the process of producing variable capital includes labor power of 20 direct from labor and 50 output from the constant capital production process. The labor value of the output of the variable capital process is 70 of the original 100 that started from the labor source. There is an



$$\text{Marx exploitation index: } e = \frac{\text{Surplus } s}{\text{Variable capital } v} = \frac{20}{20} = 1$$

Figure 6. *Systems diagram of Marx's concept of embodied labor value using his terms and numerical example of a production sector. The triangular constant gain amplifier symbol is used with a gain of 2.*

input of 20 from the environment to variable capital and regarded as surplus value. These values go to the market. In this example, 20 (equal to the surplus capital input) goes to capitalists leaving 70 returning to laborers. For steady state situations, the concept of labor value requires that the values generated from labor be considered as disappearing (used up) and being replaced anew where the pathways return labor value to laborers again.

In the example in Figure 6, the part designated as surplus, a value of 20/year, was the basis for profit and capital accumulation. Marx's rate of exploitation e is the ratio of surplus s to input variable capital v. In this case e = 1. Much of the social policy implications of Marx concerned the magnitude and use of the surplus value.

Nineteenth Century Perceptions of Money and Donor Wealth

When a commodity is traded, its inherent wealth (donor value) goes with it whether it is evaluated with hours of prior-used labor or in emergy units (prior-used energy of one form). Gold is such a commodity. An analysis of gold production in South Africa (Bhatt and Odum, 1987) suggests it has a solar transformity of 4 E14 solar emjoules per gram.

When gold is traded, it is a commodity with inherent wealth, a high emergy product of nature. Gold is used for its inherent non-corrosive properties in industry and to make jewelry.

However as Gold evolved into money, it became also a counter current to facilitate the trading of other commodities and services. Then the essence of gold was designated to paper money, gold certificates, thus circulating as counter-currents to other commodities. During the period of the last century, gold certificates could be traded for gold, a trade of information-gold in counter current for real gold. Now, in this century, we are used to money as the counter current, no longer with inherent wealth. Now the money as pure information is not useful until it is coupled to real wealth. In other words, the spending and circulating of money pumps the commodities as Keynesian economics.

Diagramming Marx's Concept of Price

Money (Information paper money, not Gold) is a counter-current to real wealth. Marx and modern economics practices often apply the numerical value of the money counter-current to the purchased commodity or service going in the opposite direction toward the consumer. The solid lines in Figures 1 and 4c represent wealth going forward and the dashed lines the counter-current of money flow. In our modern view, the ratio of money paid to commodity or service rendered in counter-current is price (not the meaning of Marx's use of the word).

In Marx's view, as we already indicated, the labor value embodied in each production process is the "cost price" which is averaged through the market to form the price of production. In Marx's way of representing the system, the pathways from labor through production to the market were measured in units of labor time (Figure 6). Beyond the market the values are represented in monetary units, "prices," moving in the direction of the products (Figure 4a).

In Marx's time, money was thought of as real value in the sense of money (gold-based) being the real wealth going with the service or commodity, rather than as a counter-current. Thus, Marx's numerical examples of flow of labor-based value, when diagrammed to represent his point of view, do not have money as a counter-current (Figure 6).

The pathways of embodied labor power are also gold except that the conversion from hours of labor to gold units was through a transformation that occurs in the market for labor that averages the labor per unit gold. See example in Figure 8. This is Marx's labor price. It is not the money paid for labor in the modern sense, but the gold-based money value that Marx sees as going in the same direction as all other commodities and, like labor power, is a way to represent the pathways from developing wealth to its consumption. Marx calls system-averaged gold-based money values "prices of production."

Figures 6 and 9 are our diagrams of Marx's model with labor value given as his gold-based money "prices." In reality, the gold and gold certificates were generally running as a countercurrent to the food, clothing, and human services and should have been represented even then as a counter-current.

Figure 7 is the example from Figure 6, with lines extending the flow of value from the market to labor and to capitalists. The evaluated price is shown dependent on cost price and in turn dependent on labor time. Marx (1963) included an arrow diagram in a letter to Engels with wages derived from variable capital and profit from surplus value. Also added to Figure 7, according to energy language diagramming, are flows of money shown in dashed lines, moving as a counter-current to the flows of labor values, although this was apparently not Marx's view.

Averaging Surplus Labor Value Through Market to Determine "Price"

In Marx's discussion, where money was assigned to a pathway numerically equal to the labor value of that flow, the number was designated the cost price. In the markets (Figure 8), the surplus part was averaged to become the profit part of the price. The commodity prices resulting have the same profit. As the diagram shows, the profit goes to capitalist, and the laborer receives the cost prices, which are less than their contributed labor value, the differences being what went into storages (not used up) and into profits.

In Figure 8, Marx's five department example (Volume III, pp. 153-155) is diagrammed to show the convergence of the several sectors, each with a different labor value and cost price input to variable capital production, each generating a different surplus value that is averaged in the market and routed on the right to capitalists as 22%. This diagram, like that in Figure 6, has surplus value generated in each department of production in excess of the 100 % input labor value.

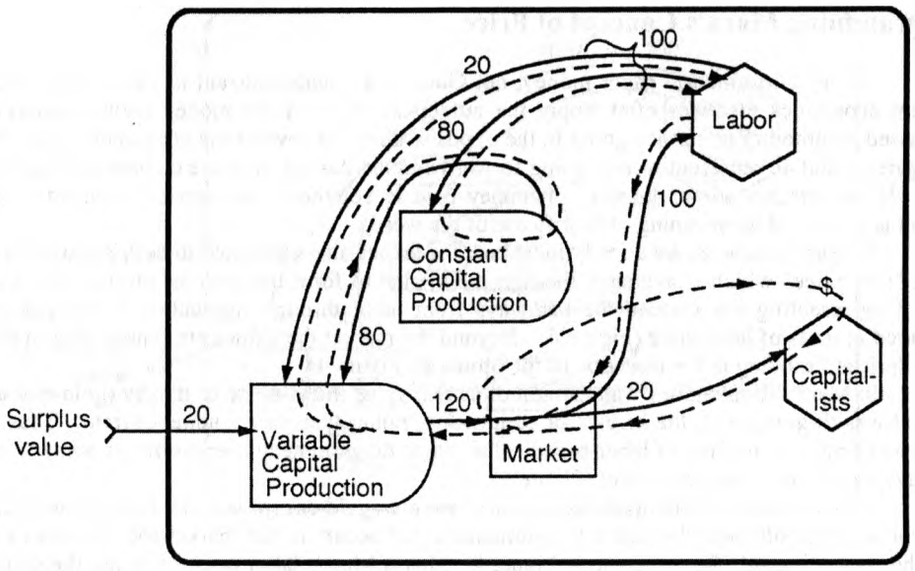


Figure 7. *Systems diagram of Marx concepts in Figure 6 extended to close production and consumption loop. Numbers are embodied labor value adjusted to steady state. Dashed lines are the flows of money in counter currents to the flows of labor and produced commodities.*

The averaging of the price of labor is tantamount to regarding laborers as interchangeable. In modern times this is done with lower paid categories with minimum wage laws. To some extent humans that are not highly educated and programmable are more flexible and thus substitutable, but human labor of our time has a wide range of educational backgrounds, job classifications, and emergy in the background of specialized human service, with workers not completely interchangeable nor representing the same price.

In the process of averaging the worker's labor inputs, an equivalence of labor and price was obtained, the price of labor power. The price of products was evaluated by the amount of the labor pool's work on the job and off the job, only part of which goes back to the workers. Foley (1986) makes Marx's conversion of labor power to wage explicit with the expression:

$$W^* = (m)\,(w)$$

where W* is the salary, which is fixed at the subsistence level, m is the value of money, and w is the money wage, the amount of money the worker received for an hour of labor power.

Closure of Producer-Consumer Loops

Figure 6 has been expanded in Figures 7 and 9 to close the inputs and outputs of the four sectors discussed: constant capital production, variable capital production, labor, and capitalists. The numbers on Figure 7 are Marx's labor values adjusted so that contributions from the labor source are numerically equal to those at the end of flows terminating at the labor sector. Whereas Figure 6 was not a steady state, values in Figure 7 were adjusted slightly to make a steady state (inflows and outflows equal for each unit).

To completely close the circuits and exchanges would require the pathways of capitalist service back to labor or other sectors, but Marx's writings did not imply such mutualism. Notice the feedback from capitalist to labor as a controller in Figure 9.

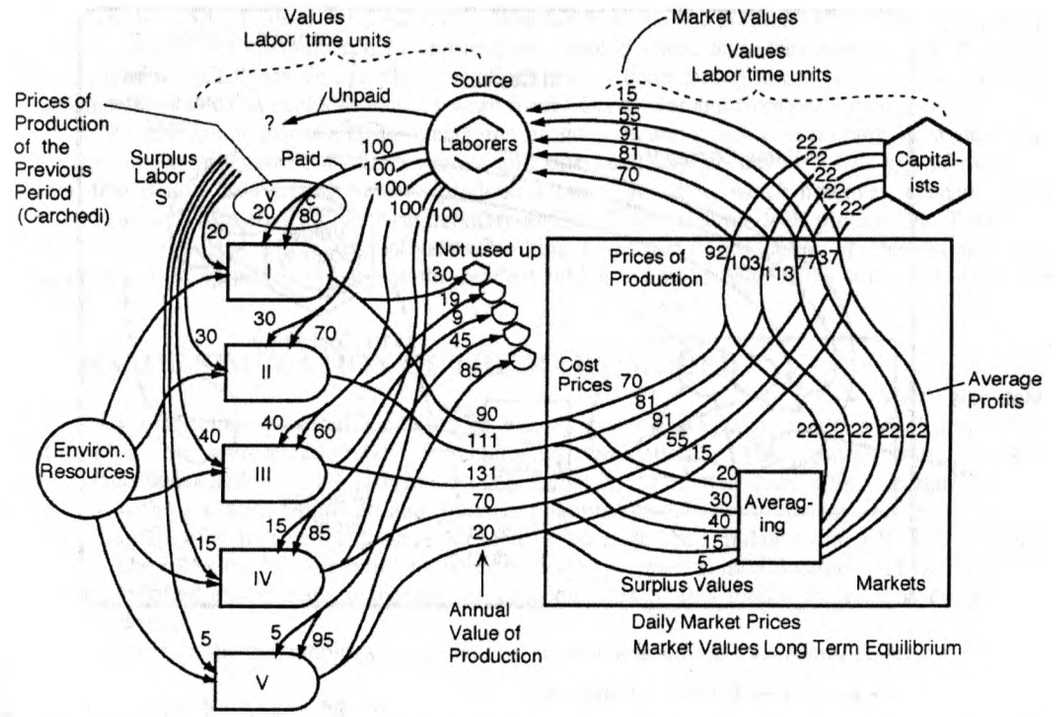


Figure 8. *Diagram of Marx's example of five production-department economy showing labor values into the market and price values for outputs from the market to consumers. Surplus capital is shown becoming profit 22% through market averaging.*

Input Output Representations of Marx Concepts of Value

Marx did not supply a complete input-output table for his examples. Several authors (Seton, 1957; Howard and King, 1976) have made partial input-output tables representing Marx's numerical examples, with three production sectors, showing surplus capital production separate from variable capital production. However, Marx rather refers to surplus capital production as an overcharge of the one system rather than as a separate sector. Neither Marx nor these later authors in their numerical tables closed the loop through the consumers (labor and capitalist) whose roles they discussed.

Diagram of Materials Flow as the Classical Circulating Capital

Table 2 has a comparison of terms simplified from Eagly (1976). Marx identified capital as consisting of two parts to which he gave two names that had been used by Adam Smith and others previously, capital for large equipment assets and capital for the materials used. In Figure 9 is given Marx's model (Figure 6) with the added detail on materials. Numerical values given are from his example from Vol. III, Chapter 9. A total labor value of 500 per unit time includes 400 circulating and variable capital and 100 constant capital. Ten of that fixed capital is "used up" in wear and tear, and thus its value is transmitted to the output commodity. (See explanation of the behavior of used up storages in Figure 5b.) Ninety is not used (If the "not-used" is stored, then the system is growing, not in steady state.) If the "not-used" is regarded as going elsewhere then the system is numerically in steady state). Average profit of 50 is generated (from source unknown) in variable capital production so that the labor value in the output commodity total is 460 per unit time of which 410 goes back to support Labor.

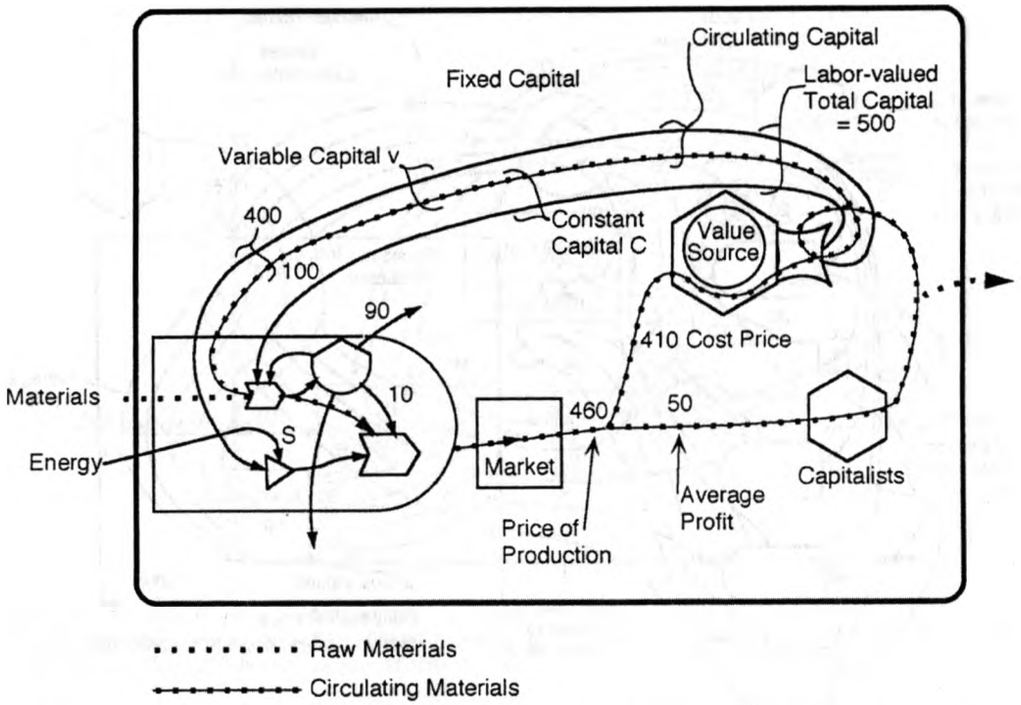


Figure 9. *Diagram of Marx's model and numerical example of labor value showing materials flow separately which Marx removed from circulating capital and considered a part of constant capital. Diagram also includes the product flows to consumers with labor-value derived prices.*

Table 2. Definitions of "Capital" by Marx Modified from Eagly (1974).

Commodity group	Some authors prior to Marx	Marx	Modern views
Wage Goods	Circulating Capital	Variable Capital	Not Usually Called Capital
Raw materials	Circulating Capital	Constant Capital	Not Usually Called Capital
Durable Producer Goods	Fixed Capital	Constant Capital	Capital

Growth Implications of Marx Model

Because surplus value in some amount above the 100% input of labor value was part of Marx's model, it was an exponential growth model. (That is, value was the input plus an added surplus increment with some production not used.) Profit was derived from the surplus. An expanding economy was generated from within a closed system of Figure 6. In 19th Century Europe, expansion was aided by the rapid incorporation of new energy sources, especially technology-facilitated fuels. The expansion of the technology to use energy reserves was generating growth in

value from the work of labor. This may have been the basis for the surplus value which Marx put in his model, sometimes attributing it to unpriced and unpaid labor, and elsewhere he said its source "remains a mystery." Marx, in one place, mentions energy contributing to production, a new concept in his time. However, he excludes nature's work from labor power and from price values:

"Natural elements entering as agents into production, and which cost nothing, no matter what role they play in production, do not enter as components of capital, but as a free gift of nature to capital, that is as a free gift of Nature's productive power to labor, which however, appears as the productiveness of capital as all other productivity under the capitalist mode of production. Therefore, if such a natural power, which originally costs nothing, takes part in production, it does not enter into the determination of price, so long as the product which it helped to produce suffices to meet the demand."

A DYNAMIC SIMULATION OF THE MARX MODEL

By diagramming verbal models with energy systems diagrams, mathematical and energy relationships are represented ready for computer simulation. The process thus combines thermodynamics, hierarchy and information, conservation of matter, reinforcing designs that are consistent with maximum empower, and dynamic relationships. In the following section, the energy systems diagram of Marx's concepts were put in equation form and simulated with a BASIC computer program. The Marx model was simulated with several experimental modifications. These "what ifs" are like controlled experiments exploring the consequences of this model for various designs and energy conditions.

Details on the methodology are given in a recent book (Odum and Odum, 2000).

Equations for the Marx Model

The Marx model in Figure 7 was redrawn without dashed money lines in Figure 10 to include the energy sources, the used energy sinks (heat sinks), and the mechanisms of production and interaction. The diagram was then translated into the equations used for simulation. The microcomputer program in BASIC for IBM PC is given in Table 3.

The state of laborers was represented as storage tank L, delivering labor power and receiving the variable capital basis for labor. This storage is the total labor including the reserve of unemployed. The use of labor in the three production interactions generates labor power. One of the labor inputs goes to variable capital production. The labor flows to the constant capital production are shown with two interactions. In some simulation runs one or the other or both are set to operate. Constant Capital Assets (A) receives products from an interaction of labor, assets, and source energy, and also from a second interaction that includes capitalist action.

The Capitalist unit C pumps value from the variable production output, diverting it from reaching Labor. In some runs, Capitalist only uses and does not feed back. In other runs, the feedbacks from Capitalist to interact and control more labor input to constant capital production is included.

Outflows of capital assets include depreciation and contribution to variable capital production, as indicated for materials-energy in Figure 5b. Variable "surplus production" was made the product of energy source, labor, and capital assets.

Provision for Connecting or Disconnecting Capitalist Action

The pathway of capitalist action is shown in Figure 10 controlling two production units that contribute products to the Assets storage. The pathways have on and off switches (X and Y). In some simulation runs a pathway process is turned off (X = 0; Y = 0), thus causing the Capitalist to take surplus capital without contributing back to the system. In other runs this capitalist-facilitated

production interaction was turned on (X 1), representing contribution and control of capitalists on some of the means of production.

Provision for Switching Unlimited and Limited Energy Sources

The importance of the type of energy source was shown in Figures 2 and 3. In the Marx simulation model in Figure 10a, provision was made to run Unlimited source (E = 0) or Flow limited source (E = 1). Both are shown in Figure 10 with an extra equation that is required for the flow limited case.

Calibration

To calibrate the model in Figure 10a, the three main storages were each given the value 100, a normalizing procedure. Then flows into and out of these storages were assigned to the pathways in proportions implied by Marx's models (Figure 6). Flows in and out were adjusted to steady state of a developed economy so as to provide an appropriate turnover time for each: Labor, 5 years; Capital Assets, 20 years; and Capitalists, 30 years. These flows and storages are shown in Figure 10b. Also shown in that Figure is the calculation of coefficients that follows from the assumed calibration values.

Simulation Results with Unlimited Energy

If the Capitalist part of the model in Figure 10 is disconnected (K7, K8, and K9 = 0), and if the source is unlimited (E = 0; R = constant), Labor and Assets and the flows among them accelerated rapidly with unlimited growth and went off scale. In other words, there is exponential growth without capitalist feedback. Some researchers have pointed out the exponential growth characteristics inherent in Marx's model. Much later, exponential growth of economies which accumulate value autocatalytically was described with equations by Domar (1937) and Harrod (1939). That simulation (Odum, 1987) bears out those conclusions about Marx's model.

Simulation with Capitalist Consuming but Not Contributing

Figure 11a has the simulation of the Marx Model without energy limit and with the capitalist consumption connected, but with feedback disconnected. Note that a pulsing oscillation results. Labor and assets started to grow and then were pulled down sharply by the capitalist consumption. A sort of prey-predator oscillation resulted.

Simulation with Capitalist Consuming and Contributing

Figure 11b has the simulation with capitalist feedback connected and energy source unlimited. The oscillation was smaller in amplitude and with higher frequency.

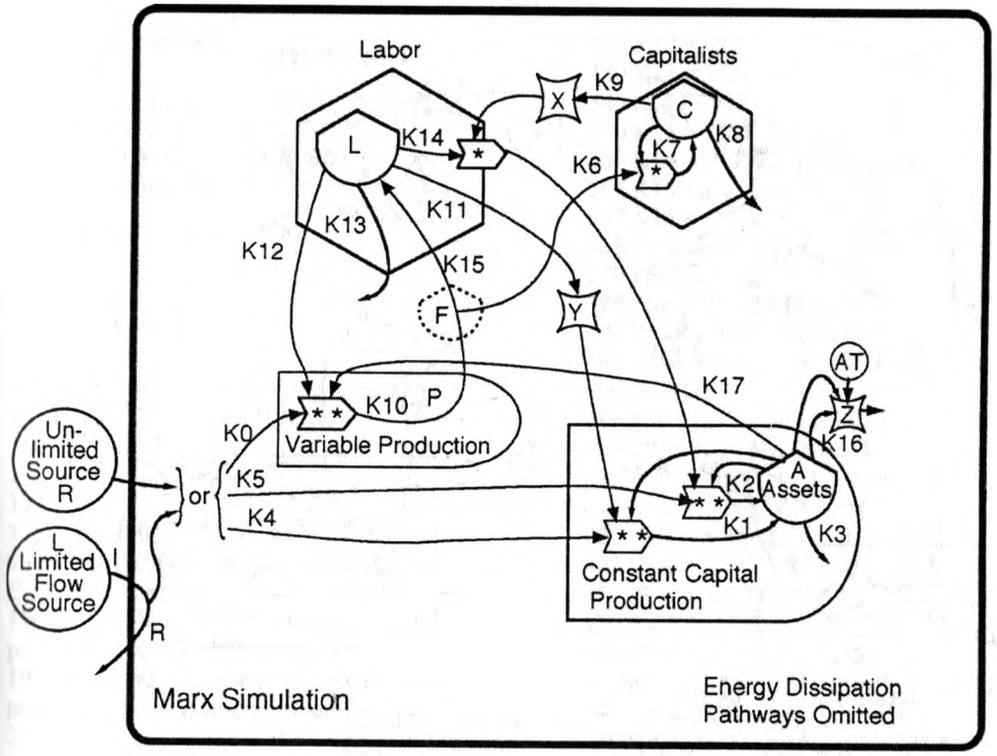
Figure 11c has the simulation with only the capitalist-facilitated production operating. The other capital assets production interaction was disabled (Y=O). There was still an oscillation.

Simulations with Limited Energy Inflow

Simulations with limited inflowing energy sources are given in Figure 12. Without the capitalist feedback connection, a broad oscillation was observed (Figure 12a). With capitalist feedback connected simulations (Figure 12b), a stable steady state pattern was observed.

Figure 12c has the simulation with only the capitalist-controlled assets production. A stable pattern with high efficiency resulted.

(a)



SWITCHES:

Type of Source = E
Renewable Source which is used by setting E = 1
R = I/(1 + k0*L*A + Y*k4*L*A +X* k5*L*A*C)
Constant, unlimited source, which is used by setting E = 0. R = I0
Assets Production with capitalist participation X = 1
Assets Production with Labor participation Y = 1
Assets Diversion when Z = 1 and A > AT

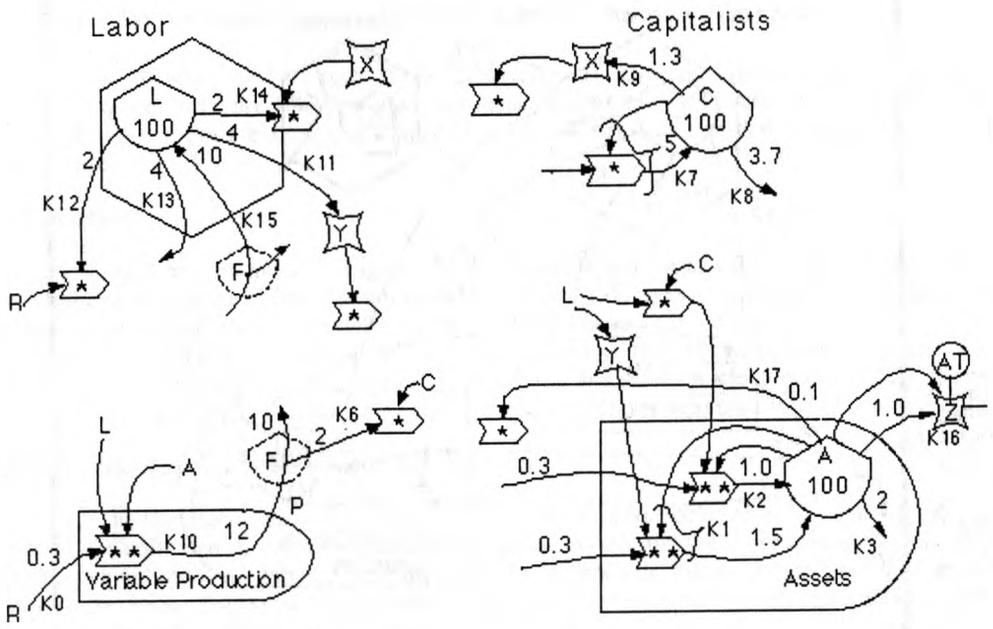
EQUATIONS:

Total Production:
PT = k10*R*L*A + Y*K1*L*A*R + X*K2*L*A*R*C

Flow to labor:
F = K10*R*A*L/(1 + K6*C)
and K6*C*F flows to capitalists

DA/dt = Y*K1*R*L*A + X2+K2*R*L*A*C - K3*A - Z*K16*A - K17*R*A*L
DL/dt = K15*F - K13*L - K12*R*A*L - Y*K11*R*A*L - X*K14*R*A*L*C
DC/dt = K7*F*C - K8*C - X*K9*R*L*A*C

(b)



Calibration values:

L = 300	I = 1.0	R*A*L = 1000
A = 100	R = 0.1	C*R*A*L = 1E5
C = 100	F = 10	F*C = 1000

Coefficients:

k0*R*A*L = 0.3	K0 = 3 E-4
Y*K1*R*A*L = 1.5	K1 = 1.5 E-3
X*K2*L*A*C*R = 1	K2 = 1E-5
K3*A = 2	K3 = 0.02
Y*K4*L*A*R = 0.3	K4 = 3 E-4
X*K5*L*A*C*R =0.3	K5 = 3 E-6
K6*F*C = 2	K6 = 2 E-3
K7*F*C = 5	K7 = 5 E-3
K8*C = 3.7	K8 = 0.037
X*K9*R*L*A*C = 1.3	K9 = 1.3 E-5
K10*R*A*L = 12	K10 = 0.012
Y*K11*R*A*L = 4	K11 = 0.004
K12*R*A*L = 2	K12 = 0.002
K13*L = 2	K13 = 2 E-2
X*K14*R*A*L*C = 2	K14 = 2E-5
K15*F = 10	K15 = 1.0
Z*K16*A = 1	K16 = 0.01
K17*R*L*A = 0.1	K17 = 1 E-4

Figure 10. *Simulation version of the Marx model (Figure 6) including the sources, storages, sinks, mechanisms, and equations. Switches are provided so as to run the model with some pathways turned on and off in different combinations. (a) System diagram with equations; (b) diagram of parts of the model with calibration values.*

Table 3. BASIC Program MARX.bas for Simulation of the Model Figure 10.

```
10 REM PC: MARX
20 CLS
30 SCREEN 1,0:COLOR 0,0
40 LINE (0,60)-(320,180),3,B
50 LINE (0,0)-(319,50),3,B
60 REM Scaling factors
70 DT = 1
80 T0 = 2
100 L0 = 5
110 C0 = 3
120 A0= 3
125 P0= 3
137 REM Sources and Starting Values
130 L = 10
140 A = 10
160 I = 2.1
170 C = 10
175 AT = 200
180 REM E is type of source: E=0 unlimited;
E=1 is flow limited
190 REM X=1 is capitalist feedback to assets
production: X=0 is disconnected
195 REM Y=1 is Labor connected to capital
production: Y=0 is disconnected
200 E = 1
210 X = 1
215 Y = 1
220 K0 = .0003
230 K1 = .0015
240 K2= .00001
250 K3 = .02
260 K4 = .0003
270 K5 = .000003
280 K6 = .02
290 K7 = .005
300 K8 = .037
310 K9 = .000013
320 K10 =.012
330 K11 = .004
340 K12 = .002
350 K13 = .02
360 K14 = .00002
365 K15 = 1!
367 K16 = .01
368 K17 = .0001
370 PSET (T/T0,180-A/A0),1
380 PSET (T/T0,180-C/C0),3
390 PSET (T/T0,180-L/L0),3
395 REM PSET (T/T0, 50 - PT/P0),1
396 GOTO 397
397 PSET (T/T0, 50 - P/P0),3
400 IF E = 1 THEN R = I/(1
+K0*L*A+Y*K4*L*A + X*K5*L*A*C)
410 IF E = 0 THEN R = 2.1
420 IF R < .0001 THEN R = .0001
425 IF A > AT THEN Z =1
426 IF A < AT THEN Z =0
430 P = K10*R*L*A
435 PT = P +Y*K1*L*A*R
+X*K2*L*A*R*C:REM total production
440 F = K10*R*L*A/(1 + K6*C)
450 DA=Y*K1*L*A*R +X*K2*L*A*R*C -
K3*A - Z*K16*A - K17 * R*A*L
460 DC=K7*F*C - K8*C -X*K9*R*L*A*C
470 DL = K15*F - K13*L -K12*R*A*L-
Y*K11*R*A*L -X*K14*R*A*L*C
480 C = C +DC*DT
490 IF C <1 THEN C = 1
500 A = A + DA * DT
505 IF A <1 THEN A = 1
510 L = L +DL*DT
515 IF L <1 THEN L = 1
520 T = T + DT
530 IF T/T0<320 GOTO 370
```

Comparison of Simulation with 19th Century Conditions

In the 19th Century, with new resources being drawn into the economy with new technologies, conditions were like the simulations with unlimited energy. The sharp economic oscillations observed in that century may be consistent with the simulation oscillations.

Like Domar (1937) and Harrod (1939) exponential growth formulation much later, it was regarded as normal that surplus, profit, and expanding economy could be generated from within a closed system of Figure 6. Such change in the 19th Century was started by the rapid incorporation of new sources, especially technology-facilitated fuels.

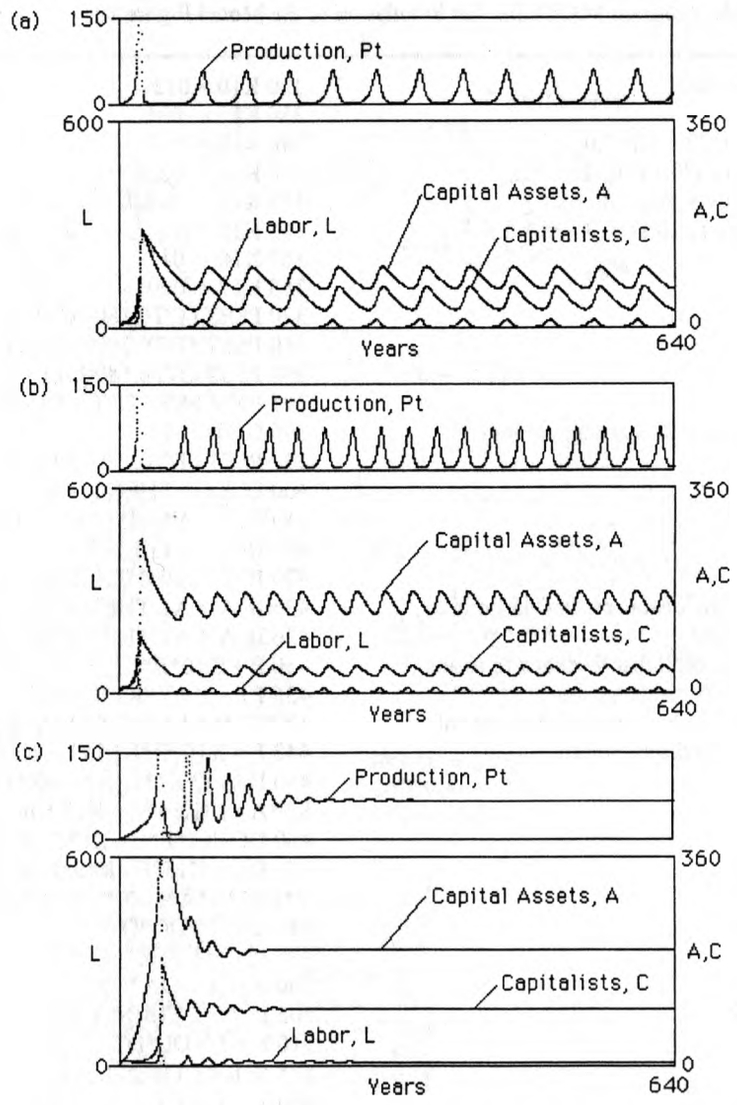


Figure 11. *Simulations of the model in Figure 10 with unlimited energy source (E = 0; R = constant): (a) Without feedback from capitalist growth (X = 0; Y = 1); (b) with some production aided by feedback from capitalist growth (X = 1; Y = 1); (c) with the only production controlled by feedback from capitalist growth (X = 1; Y = 0).*

Especially when the capitalist sector contribution was not connected, the simulations generated less production than was possible, and so little was routed to laborers that their services were limiting to economic vitality.

Many of the discussions of this system have concerned what happens when the circulation in Figure 10 is growing, with the surplus loop generating a new storage of accumulated capital (money). Following Marx, Figure 10 shows the accumulated capital going into luxury production for capitalists, who cause labor to produce a surplus for this. The words "luxury and surplus" used by Marx and the diagram representing his views (Figure 10), were implied criticisms of the system of his time.

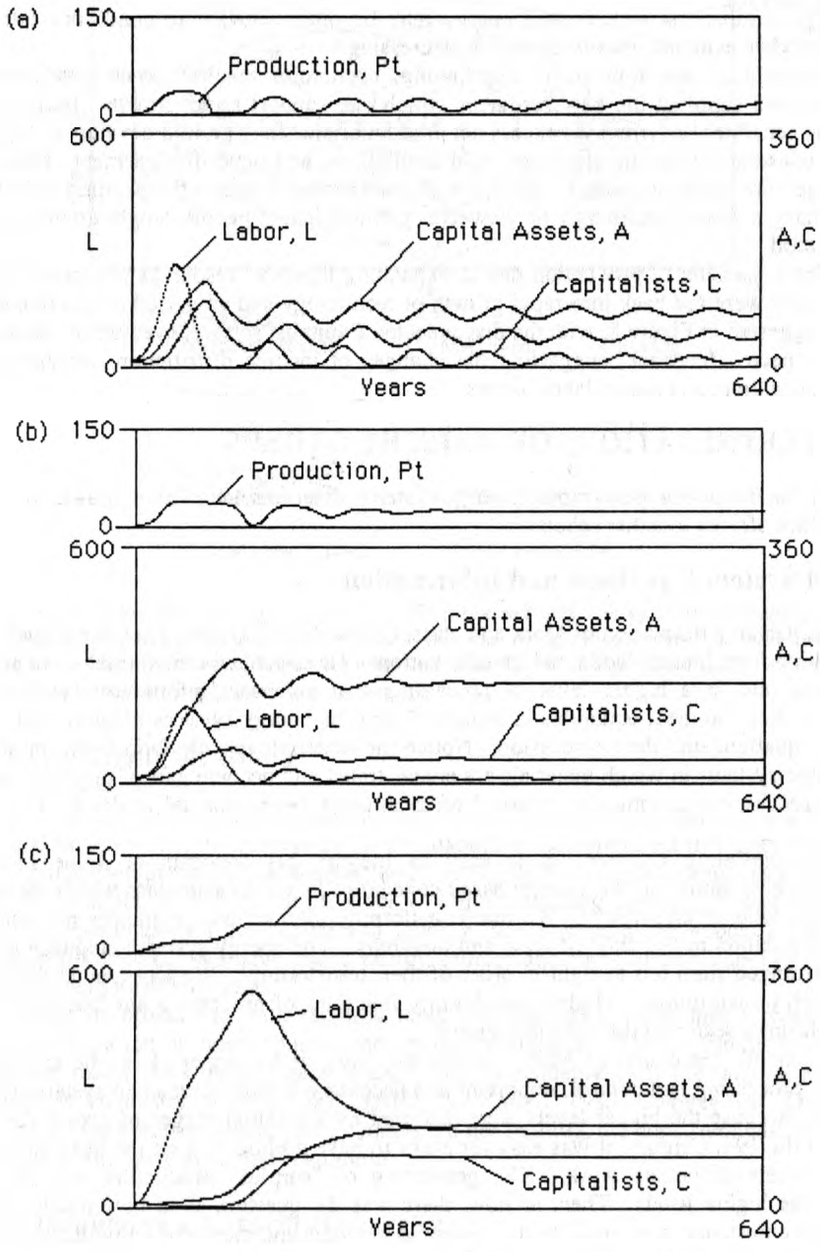


Figure 12. *Simulations of the model in Figure 10 with energy sources limited to a fixed inflow (E=1; R limited). (a) Without any production feedback action from capitalist growth (X = 0; Y = 1); (b) with some production controlled by the capitalist feedback (X = 1; Y = 1). (c) with limited source and with all the production controlled by capitalist feedback (X = 1; Y =0).*

Comparison of Simulation with 20th Century Conditions

The simulations with limited energy may be more relevant to conditions late in the new millennium when available resources may be decreasing.

Incomplete, and thus badly functioning, economies resulted when low energy agrarian economies were supplied the new resources which can support rapid growth. Early succession in ecosystems generates patterns with excess net production that may go into developing higher levels of voracious consumers with low diversity, wild oscillations, and rapid displacement of the occupations of one stage with those of another. Thus, the disruption and change of expanding resources that are self organizing a better system may be viewed as pathological to people caught up in change that was not understood.

The expansion of production due to expanding inputs of resources generated surplus values that eventually were fed back into rapid growth of technology and advanced production sectors. The structure suggested in Figure 8, with the disconnected Capitalist service, eventually was adjusted to be like the connected feedback, suggesting the changes of income distribution, reorganization of the economy, education, and rise of labor unions.

THE INTERPRETATIONS OF MARX BY OTHERS

In the following paragraphs, energy systems diagrams have been drawn to represent the views of Marx offered by other scholars:

Carchedi System Equations and Information

As part of a discussion of Marx and class, Carchedi (1987) gave a set of equations describing value resulting from human-facilitated transformations. He described transformation in production, in consumption, and in a higher level of processing that generates information, and finally social information. See Carchedi equations in Figure 13 and the energy systems diagram that is consistent with these equations and their discussion. Notice the materials recycle loop found in all ecological and economic systems in which materials are concentrated and become part of upgraded products and then dispersed by consumption as "waste," often through environmental cycles back to production again.

By translating Carchedi, it is easy to identify his concepts of value resulting from transformations as similar to the emergy based concept of hierarchy and class which we have offered (Odum 1986; Scienceman, 1987). We use Transformity, defined as the emergy per unit energy, to measure the position in the scale of class and hierarchy. The energy systems language has units and processes arranged from left to right in order of their transformity. We have found that information has very high transformities. Higher transformity functions of processing and control are identified with the consumer sector on the right in Figure 8.

Carchedi's discussion of Marx inserted the work of the upper class, the mental labor and information processing and control, important and necessary in fully developed systems (right side of Figure 13). Because the higher levels were distorted by the initial surges of growth based on new resources in the 19th Century, it was easy for Marx to have a blind spot on the need and necessity of the higher levels of human work. The generating of "surplus" production was the means for developing the higher levels. Then, as now, there was the question as to how much of the luxury, pomp, and circumstance is essential to the system as symbols and means of individual motivation, and how much is pathological diversion leading to economic failure in circumstances of world competition.

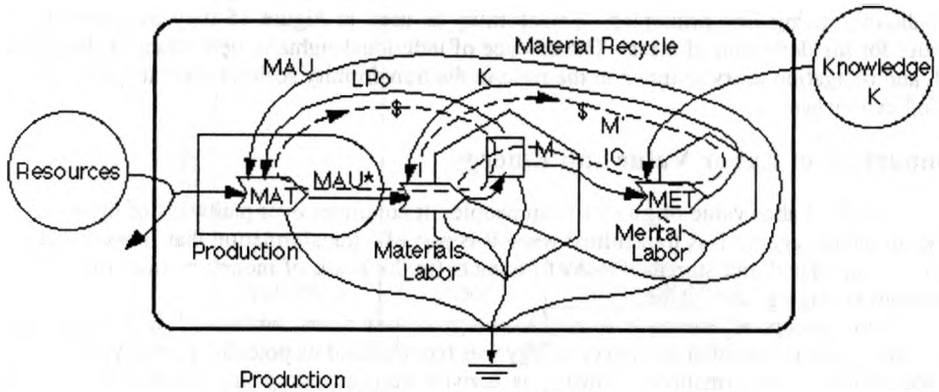


Figure 13. *Transformation equations of Carchedi (1987) expressed with energy systems language. Carchedi's expressions: LPo, labor power in value production; MAT, material transformation; MET, mental transformation; MAU, material use value; MAU*, transformed material use value; K, knowledge; K*, transformed knowledge; IC perception of concrete reality used to transform knowledge.*

Samuelson Equations for Marx

A set of linear programming equations was used by Samuelson to represent the Marx model of Figure 6 and to study analytic properties. His equations are diagrammed in Figure 14. They are similar to the production part of the simulation model in Figure 10. One production system (I) sums a contribution from capital K and one from labor L while producing capital. A second production system II sums contributions from capital and labor while generating the goods Y that are used (transformed) into Labor L. We have added energy sources and sinks (dotted lines) to Figure 14 to make the model energetically correct.

A system of production with required labor and capital, if made into a dynamic model, requires the multiplication of these inputs as the production function. The configuration of two such systems with two interactions looping back to each other is mathematically the same as an explosive chain reactions in chemistry (Figure 14a). (Also, see Odum, 1983, Chapter 6.) The explosive properties of this configuration, where energy inputs are not limiting, are consistent with the explosive nature of growth in the early industrial revolution.

Process of Organizing a Natural Hierarchy

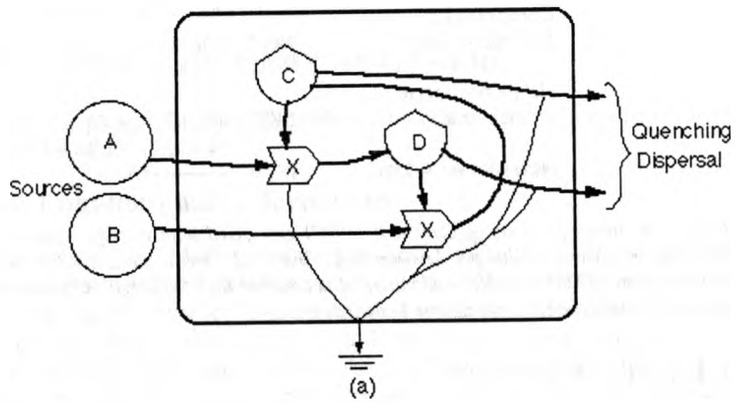
We have indicated elsewhere the natural energy-hierarchy which emerges with successful self organization of many kinds of systems, animate and inanimate. Systems of different kinds are compared by expressing each on a graph of Energy and Transformity (Figure 15). The struggles over distribution and class described as dialectical by Marx seem to be the means by which self organization finds the natural distribution. If the natural hierarchical energy-transformity pattern predicts the outcome of social-political evolution, we have means for achieving a stable economy

more directly, using first principles. Transformity as used in Figure 15 may be a useful, objective measure for the definition of class. The essence of individual rights in democracy is the opportunity, right, and obligation to try to move to the right in the transformity scale of class in the course of one's life and experience.

Comparison of Labor Value and Emergy

Marx's Labor value originates from people. It continues over pathways of the system until it returns to people again. It is transmitted when it is part of a transformation that "uses up" an input. It may be accumulated and stored. Price of products (in the sense of monetary value) was believed to come from averaging labor value.

Our concept of emergy is available energy of one form "embodied" in a commodity or in labor. Emergy is transmitted whenever energy was required and its potential partially used up in being degraded during transformations. Emergy is derived from outside energy sources or from storages that



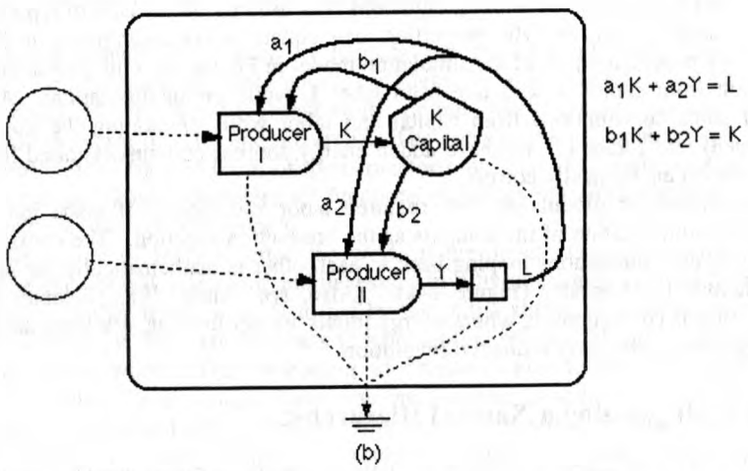


Figure 14. *Energy diagram of Samuelson's (1957) linear equations for representing Marx and a comparison with chain reactions. (a) Energy diagram of explosive chain reactions of unlimited energy sources in reactants A and B generating products D and C; (b) energy diagram of Samuelson's equations for Marx. Dotted lines were used to add energy flows.*

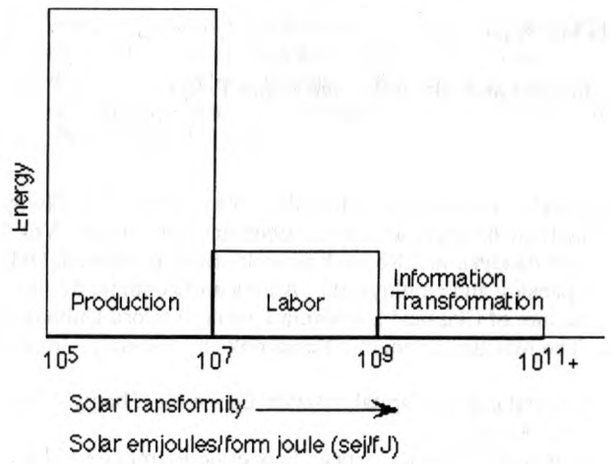


Figure 15. *Energy-transformity diagram used to represent class hierarchy.*

were derived from outside previously. Emergy values disappear when a pathway returns to amplify production in a closed loop. Labor has an emergy value, but the amount depends on the type of labor, which in turn depends on the emergy of inputs to that labor such as education and machinery used. If one gives labor and services an emergy value in proportion to wages, one is using an average money per unit value, a useful averaging procedure for some purposes. Both are donor embodied values, but emergy evaluates all flows with numbers derived from scientific data on the quantity of energy previously used. Marx's numerical values for a flow are the labor hours contributed.

Trade Equity

Lonergan (1988) compared several ways of evaluating the relative benefits to each of two trading partners. Becker (1976) had used Marx's labor value concept to show large imbalances where trade was based on market prices. We had shown large imbalances using emergy (Odum, 1984; Odum and Odum, 1983; Odum, 1988). If labor value is used to show a low price of labor in rural areas, the implication might be drawn that labor is exploited there. The emergy evaluation indicates a different interpretation. Labor may have as high a standard of living in units of emergy per person. Emergy values for products from rural countries in relation to price are higher than in developed countries because more of the support of labor comes direct from the landscape without payment. In both evaluations, the country receiving higher values exploits the other unless it returns to that country balancing values of other kinds such as information, technology, military protection, education, health programs, foreign aids, etc.

Maximum Emergy and Economic Structure

Marx was a pioneer in seeking a donor basis of value. However, by inadequately recognizing the basis for labor value in external inputs from emergy sources that were increasingly available in his time, he misinterpreted the meaning, hierarchical distortion, trends, and remedies.

The world still has problems finding appropriate hierarchical structure that reinforces and stabilizes economies to maximize total system performances. Maximum performance may require a hierarchical distribution of emergy among classes of service-contributing consumers. The issue still is to redivert the output of production from unnecessary luxury and waste to feed back reinforcements to production.

ACKNOWLEDGMENT

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